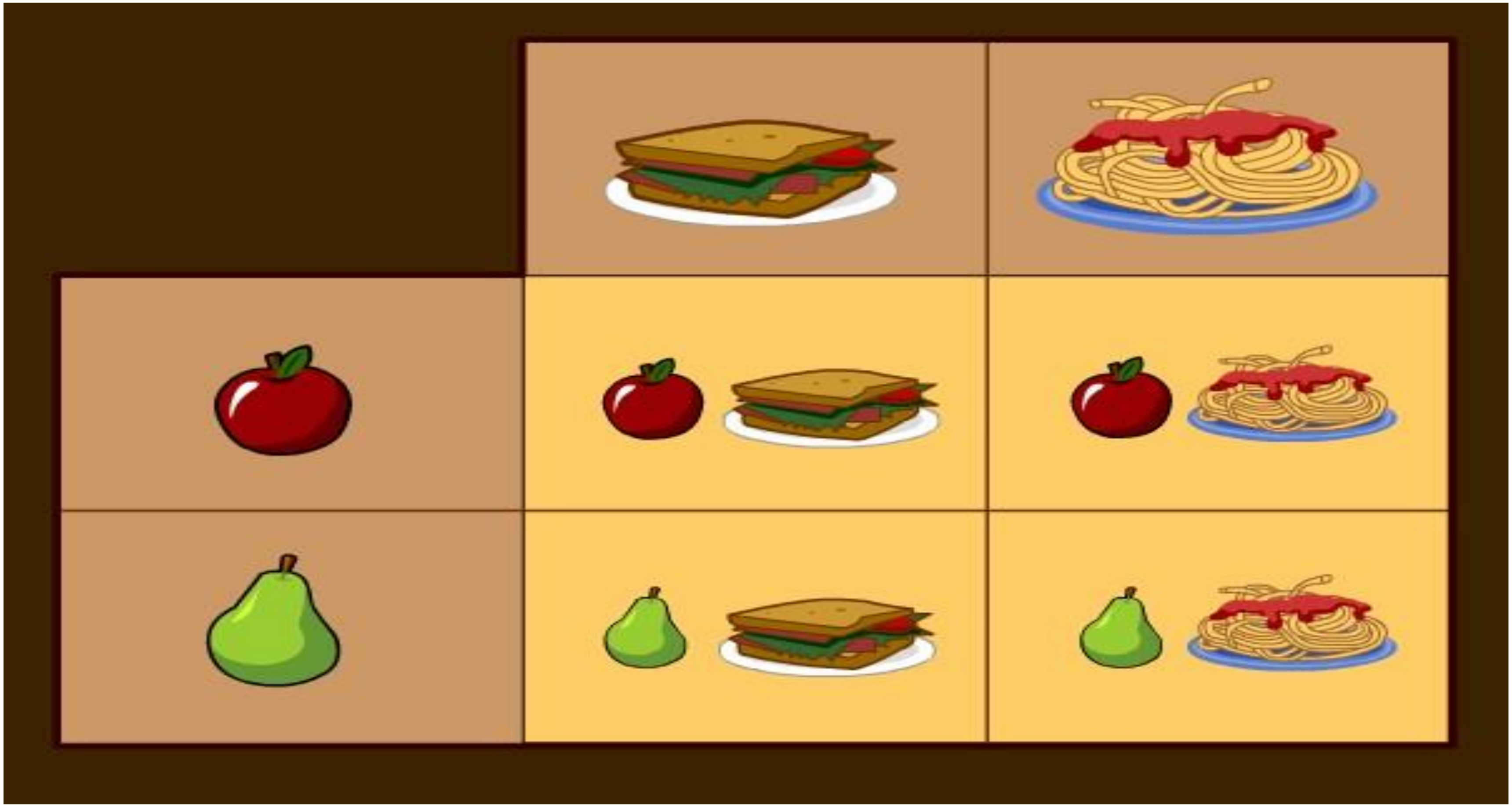
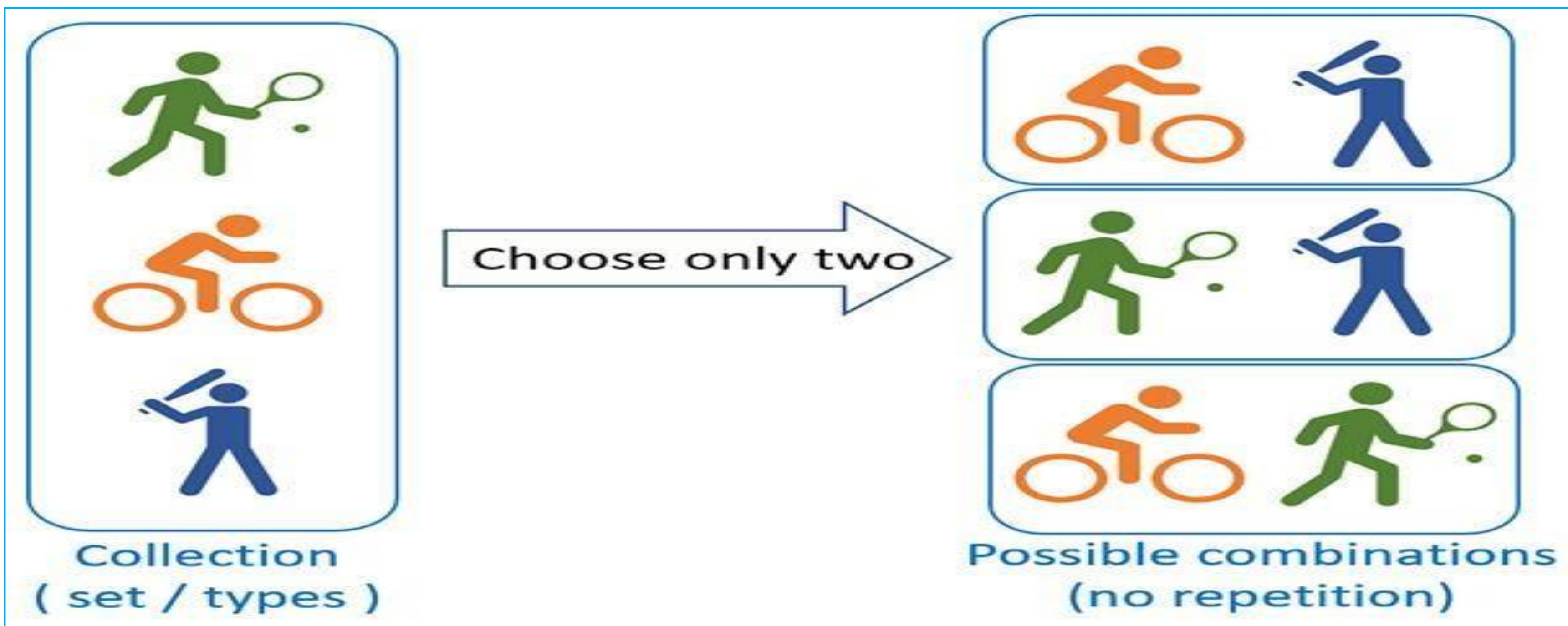
COMBINATIONS

MODULE-6



LET'S LOOK AT AN EXAMPLE......

Here is a visual example of how combinations work. Say you have to choose two out of three activities: Cycling, Baseball and Tennis. The possible combinations would look as shown:



Let’s see a general case:

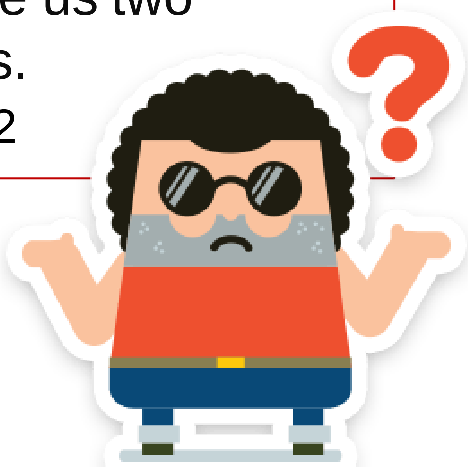
* In English we use the word "combination“, without thinking if the **order** of things is important.
* ***"My fruit salad is a combination of apples, grapes and bananas"*** We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", its the same fruit salad.
* ***"The combination to the safe is 472"***. Now we **do** care about the order. "724" won't work, nor will "247". It has to be exactly **4-7-2**.

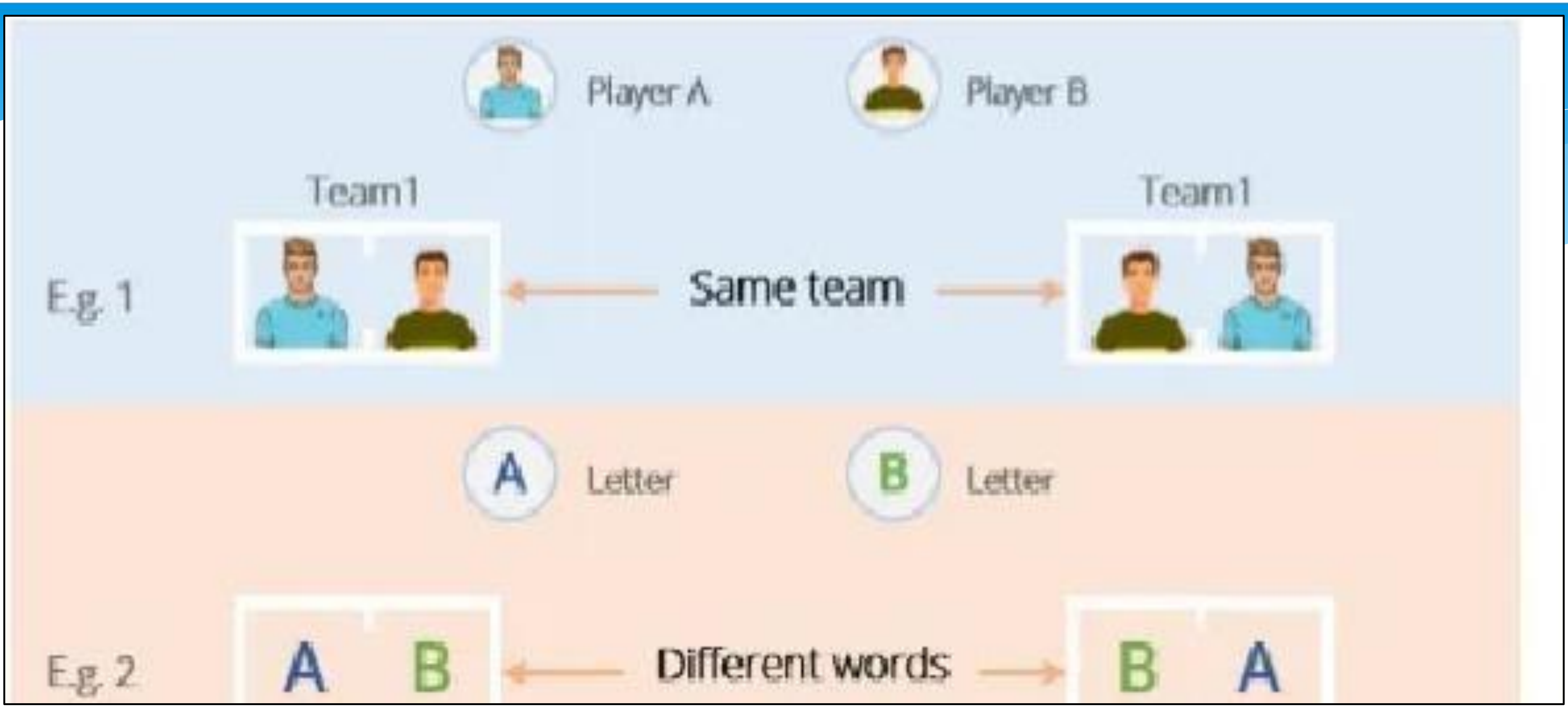
* So, in Mathematics we use more *precise* language:
* When the order doesn't matter, it is a **Combination**.
* When the order **does** matter it is a **Permutation.**

What's the Difference?

From 3 players A, B and C, how many doubles team can be formed?	**From 3 letters A, B and C, how many 2-LETTER words can be formed?**
The team (AB) is same as the team (BA).	The word AB is not same as word BA.
Arrangement of the team members does not affect the team composition.	The arrangement of the letters can give us two different words.
3	**6** $={}^{3}P_{2}$

Do both the examples look the same to you?
Well….the examples are not the same !





From 3 players A, B and C, the teams of two players can be:
1) Team AB   2) Team AC   3) Team BC.
Thus we can form 3 teams. This is a case of combination.

The 2-letter words that can be formed from 3 letters A, B and C are:
1) AB   2) BA   3) AC   4) CA   5) BC   6) CB
Thus we can form 6 different words. This is a case of permutation.

SUPPOSE THERE ARE 4 OBJECTS , TAKEN 2 AT A TIME

A, B, C , D

SELECTION	ARRANGEMENT
AB	AB, BA
AC	AC,CA
AD	AD,DA, BC, CB
BC	BD,DB
BD	CD,DC
CD	12 arrangements= $^4p_2$
6 combinations	

ARRANGEMENT = COMBINATION X 2!

ARRANGEMENT = COMBINATION X 3!

So, from the above example, can you see that permutation is same as doing selection first and then doing arrangement??

That is, $\boxed{{}^{3}P_{2} = {}^{3}C_{2} \times 2\,!}$

THINK !

Let us understand this mathematically:

$$^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{n!}{(n-r)!\times r!} \times r! = {}^{n}C_{r} \times r!$$

Thus, we can simply write:

Permutation of 'r' things from 'n' things	=	Selection of 'r' things from 'n' things	✖	Arrangement of all the selected 'r' things

Hence, permutation = $\boxed{{}^{n}C_{r} \times r!}$

Combinations

A combination is a grouping or subset of items.
For a combination, the order does not matters.

$$C(n, r) = {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

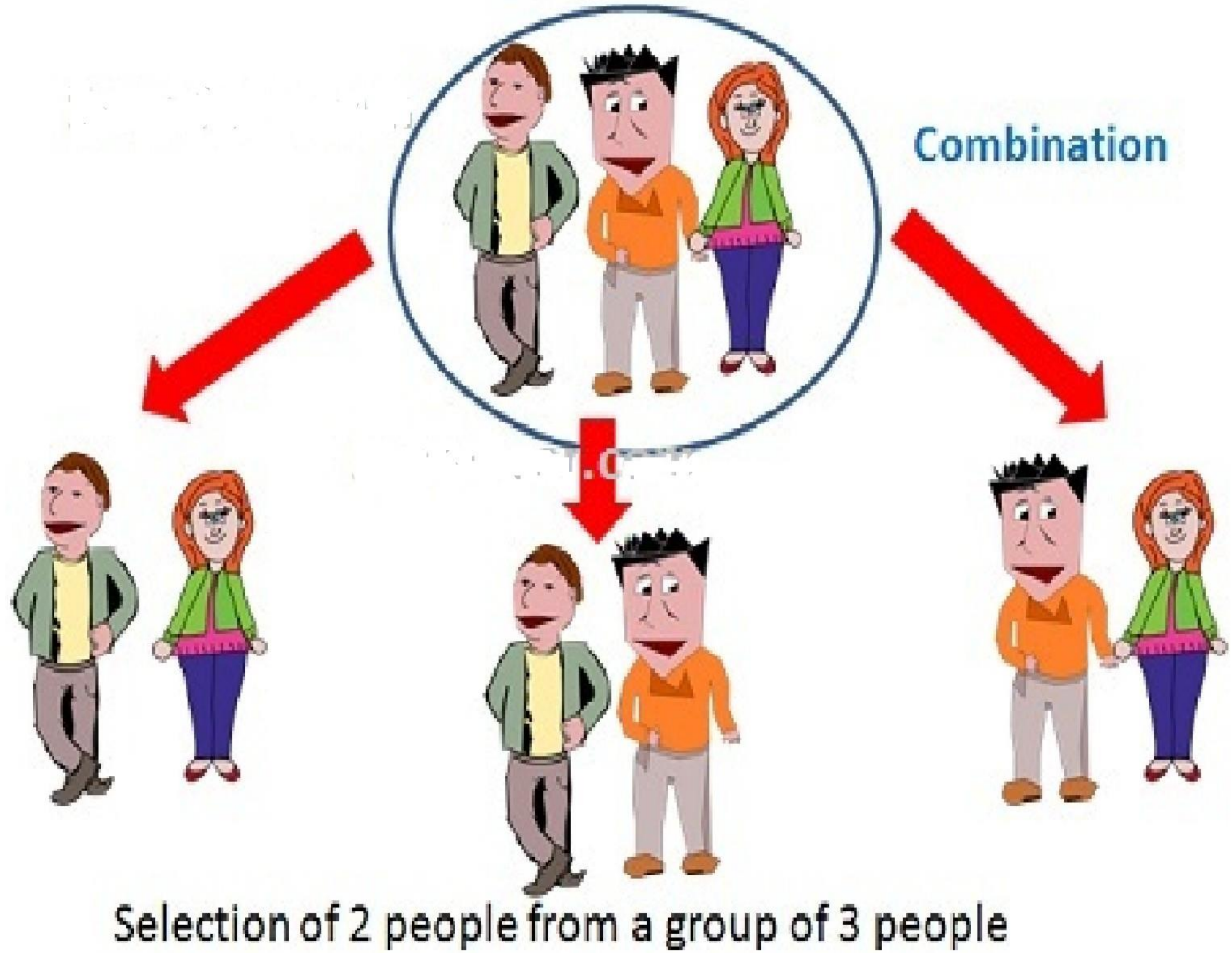
Number of items in set

Number of items selected from the set

NOTE: (i) The number of combinations(selections) of n different things taken all at a time is 1. That is, $\boxed{{}^{n}C_{n} = 1}$

(ii) The number of combinations(selections) of n different things taken none at a time is 1. That is, $\boxed{{}^{n}C_{0} = 1}$

LET US CONSIDER A SITUATION ……..



As we know that the number of combinations of n different objects taken r at a time is given by

$$ {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!} $$

From the given situation…..

When 2 persons are selected from a group of 3 persons, in each of these selections, one person is left out and this happens in 3 ways.

So, $\boxed{{}^{3}C_{2} = {}^{3}C_{1} = 3}$

If out of n things, we take r at a time, then automatically, we are left with a group of $(n-r)$ things. Hence,

$$\begin{pmatrix}\text{the number of combinations} \\ \text{of n things taken r at time}\end{pmatrix} = \begin{pmatrix}\text{the number of combinations of} \\ \text{n things taken (n − r) at time}\end{pmatrix}$$

$$\Rightarrow \boxed{{}^{n}C_{r} = {}^{n}C_{n-r}}$$

If 'n' is given and 'r' is not known….

If, $\boxed{{}^{n}C_{r} = {}^{n}C_{n-r}}$

That is, $\boxed{{}^{n}C_{a} = {}^{n}C_{b}}$ $\Rightarrow$ Either $a = b$

Eg: $\boxed{{}^{10}C_{r} = {}^{10}C_{10-r}}$ $\Rightarrow$ $r = 10 - r \Rightarrow 2r = 10 \Rightarrow r = 5$

If 'n' is not given and 'r' is known…..

Or $a + b = n$

Eg: $\boxed{{}^{n}C_{2} = {}^{n}C_{8}}$ $\Rightarrow n = 2 + 8 = 10$

Let's Practice……

Q 1 – In a society of 10 members, we have to select a committee of 4 members. As the owner of the society, John is already a member of the committee. In how many ways the committee can be formed.

Solution: **We are asked to select a committee of 4 members from 10 members and John is already a part of the committee. Thus, we have to select 3 members among 9.**

We can select 3 members from 9 members in $9C_3$ ways

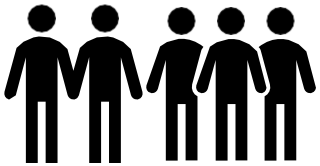
which is equal to $\frac{9!}{6!\times 3!}$ = 84 ways.

Continued……

Q2)In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

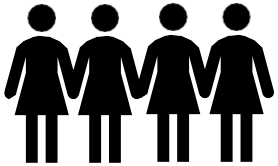
Answer. A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in $^5C_3$ ways.



3 girls can be selected from 4 girls in $^4C_3$ ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$



$= \frac{5\times4\times3!}{3!\times2} \times \frac{4\times3!}{3!}$

$= 10 \times 4 = 40$

Q3)In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Answer. Out of 17 players, 5 players are bowlers.
A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.
4 bowlers can be selected in ${}^{5}C_{4}$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_{7}$ ways.
Thus, by multiplication principle, required number of ways of selecting cricket team

$$={}^{5}C_{4}\times{}^{12}C_{7}=\frac{5!}{4!1!}\times\frac{12!}{7!5!}=5\times\frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1}=3960$$

Q4)A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Answer. There are 5 black and 6 red balls in the bag.
2 black balls can be selected out of 5 black balls in ${}^{5}C_{2}$ ways and 3 red balls can be selected out of 6 red balls in ${}^{6}C_{3}$ ways.
Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls

$$={}^{5}C_{2}\times{}^{6}C_{3}=\frac{5!}{2!3!}\times\frac{6!}{3!3!}$$

$$=\frac{5\times4}{2}\times\frac{6\times5\times4}{3\times2\times1}=10\times20=200$$

ASSIGNMENT QUESTIONS…..

Q1)A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if (a) they can be of any colour (b) two must be white and two red and (c) they must all be of the same colour.

Q2)In how many ways can a football team of 11 players be selected from 16 players? How many of them will

(i) include 2 particular players?

(ii) exclude 2 particular players?

Q3)A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls.

Q4) A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw.

Q5) A student has to answer 10 questions, choosing atleast 4 from each of Parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

Q6) Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Q.7) In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

Q8) In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are

Q.9) We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?

Q.10) How many committee of five persons with a chairperson can be selected from 12 persons.

ANSWERS:

1) (i) 330 (ii) 150 (iii) 20 2) 4368 (i) 2002 (ii) 364

3) (i) 21 (ii) 441 (iii) 91 4) 64 5) 266 6) 2000

7) 3 8) 35 9) 22 10) 3960


Thank You and Happy Learning
STAY
HOME
STAY
SAFE